上海交通大學
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Institute of Media, Information, and Network

Chapter 3 Fourier Series Representation of Periodic Signals

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Topic

- 3.0 Introduction
- 3.1 The Response of LTI Systems to Complex Exponentials
- 3.2 Fourier Series Representation of Continues-Time Periodical Signals
- 3.3 Convergence of the Fourier series
- 3.4 Properties of Continues-Time Fourier Series
- 3.5 Fourier Series Representation of discrete-Time Periodical Signals

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- If we know the response of an LTI system to some inputs, we actually know the response to many inputs

If $$x_k[n] \rightarrow y_k[n]$$

then $$\sum_k a_k x_k[n] \rightarrow \sum_k a_k\, y_k[n]$$

- If we can find sets of "**basic**" signals so that
  - We can represent rich classes of signals as linear combinations of these building block signals.
  - The response of LTI Systems to these basic signals are both simple and insightful.

Candidate sets of basic signal

- Time domain $\delta(t)/\delta[n]$

① $x(t)=\int_{\tau} x(\tau)\delta(t-\tau)d\tau$ $\quad$ $x(n)=\sum_{k} x[k]\delta[n-k]$

② $\delta(t)\rightarrow h(t)$ $\quad$ $\delta[n]\rightarrow h[n]$

$x(t)\rightarrow y(t)=x(t)*h(t)$ $\quad$ $x[n]\rightarrow y[n]=x[n]*h[n]$

Candidate sets of basic signal

- Frequency domain $e^{j\omega t} / e^{st}$

① $x(t) = \int_{\tau} x(\tau)\delta(t-\tau)d\tau \qquad x(t) = \int_{\omega} ?\, e^{j\omega t} d\omega$

② $\delta(t) \to h(t) \qquad e^{j\omega t} \to H(j\omega)$

$x(t) \to y(t) = \int x(\tau)h(t-\tau)d\tau \qquad x(t) \to y(t) = \int_{\omega} ?\, H(j\omega)d\omega$

$= x(t) * h(t)$

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The signal is decomposed into a linear combination of elementary signals

- The basic signal shall satisfy:
  - Can represent a fairly broad class of useful signals with the "linear combination" of the basic signals.
  - The response of the LTI system to the base signal should be very simple, and the response of the system to any input signal may be conveniently represented by the response of the base signals.

Complex exponential signal as basic signal

1. Representation

$$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}X(s)e^{st}ds,\quad X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt$$ ——Laplace变换

$$x[n]=\frac{1}{2\pi j}\oint X(z)z^{n-1}dz,\quad X(z)=\sum_{n=-\infty}^{\infty}x[n](z)^{-n}$$ ——z变换

2. Response of LTI

$$x(t) = e^{st} \quad y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}d\tau = e^{st}\int h(\tau)e^{-s\tau}d\tau$$

Let

$$H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau$$

——System function, assuming it is converged

then

$$y(t) = H(s)e^{st}$$

The response to complex exponential of the LTI system:

$$e^{st} \to H(s)e^{st}$$

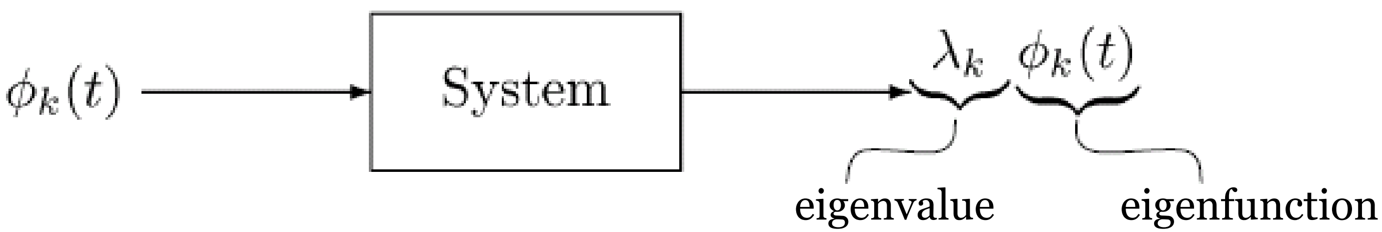
the complex exponential with the same frequency, but scaled with H(s)

Similarly, for DT systems, one obtains

$$z^n \to H(z)z^n$$

The response of a LTI system to a complex exponential is a complex exponential with the same frequency, but scaled with H(s)/H(z).

$$e^{st} \rightarrow H(s)e^{st} \qquad z^n \rightarrow H(z)z^n$$



e^{st} / z^n —— **eigenfunction (特征函数)**

$H(s)/H(z)$ —— **eigenvalue (特征值)**

Following Eigenfunction property and superposition property of LTI systems, one obtains:

$$x(t) = \sum_k a_k e^{s_k t} \rightarrow y(t) = \sum_k a_k H(s_k) e^{s_k t}$$

$$x[n] = \sum_k a_k (z_k)^n \rightarrow y[n] = \sum_k a_k H(z_k)(z_k)^n$$

If the input to an LTI system is represented as a linear combination of complex exponentials, the output will be the linear combination of complex exponentials, each part is weighted by $H(s_k)/H(z_k)$, i.e. the weighted value is depending on the frequency response associated with of the exponential component (s_k/z_k) .

Periodical complex exponential signal as basic signal

$$e^{st} \xrightarrow{\mathrm{Re}[s]=0} e^{j\omega t}$$

$$(z)^{n} \xrightarrow{|z|=1} e^{j\omega n}$$

- Decompose signal as a linear combination of e^{jwt} / e^{jwn} , and find out the response of the signal based on the response of e^{jwt} / e^{jwn} .

------ The Fourier Transform

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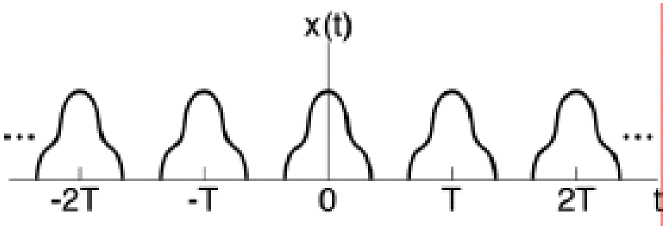
Fourier Series Representation of CT Periodic Signals

The Fourier transform of a continue time periodic signal is that the continue time periodic signal is represented by a linear combination of a group of harmonic signals or sinusoidal signals. Mathematically, they are a complete set of orthogonal functions.

$$x(t) = x(t+T) \quad \text{for all } t$$



-smallest such *T* is the *fundamental period*

- is the *fundamental frequency*

$$e^{j\omega t} \text{ periodic with period } T \Leftrightarrow \omega = k\omega_0$$

$$\Downarrow$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi t/T}$$

Question #1: How do we find the Fourier coefficients? (Given $x(t)$, how find a_k?)

1) multiply by $e^{-jn\omega_0 t}$
2) Integrate over one period

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

1) multiply by $e^{-jn\omega_0 t}$
2) Integrate over one period

$$\Downarrow$$

$$\int_T x(t) e^{-jn\omega_0 t} dt = \int_T \left(\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \right) e^{-jn\omega_0 t} dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \left(\int_T e^{j(k-n)\omega_0 t} dt \right)$$

(Here $\int_T$ denotes integral over any interval of length T (one period).)

Next, note that

$$\int_T e^{j(k-n)\omega_0 t} dt = \begin{cases} T, & k = n \\ 0, & k \neq n \end{cases}$$

$$= T\delta[k-n]$$ Orthogonality

$$\Downarrow$$

$$\int_T x(t)e^{-jn\omega_0 t}dt = \sum_{k=-\infty}^{\infty} a_k \left(\int_T e^{j(k-n)\omega_0 t}dt\right) = \sum_{k=-\infty}^{\infty} a_k \cdot T\delta[k-n]$$

$$\int_T x(t)e^{-jn\omega_0 t}dt = a_n T$$

$$\Downarrow$$

CT Fourier Series Pair $\left(\omega_0 = \frac{2\pi}{T}\right)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$ (Synthesis equation)

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$ (Analysis equation)

Fourier Series Representation of CT Periodic Signals:

(1) The periodic signal x(t) could be constructed as a linear combination of the harmonically related complex exponentials (sinusoidal signals)

(2) a_k is represents the magnitude and phase of k^{th} harmonic component

(3) $\{a_k\}$ are called as Fourier series coefficients, or spectrum of x(t)

$\{|a_k|\}$ -- magnitude spectrum, $\{\arg(a_k)\}$ --phase spectrum

Example:

$$a_0 = \frac{1}{T}\int_T x(t)dt$$

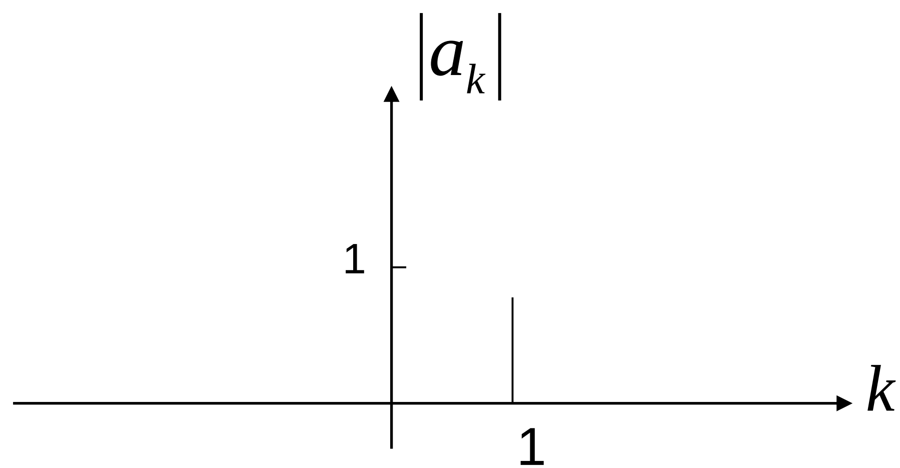
—constant component or DC component of x(t)

Example:

$$x(t) = e^{j\omega_0 t}$$

$$a_1 = 1$$

$$a_k = 0, \mathrm{k} \neq 1$$

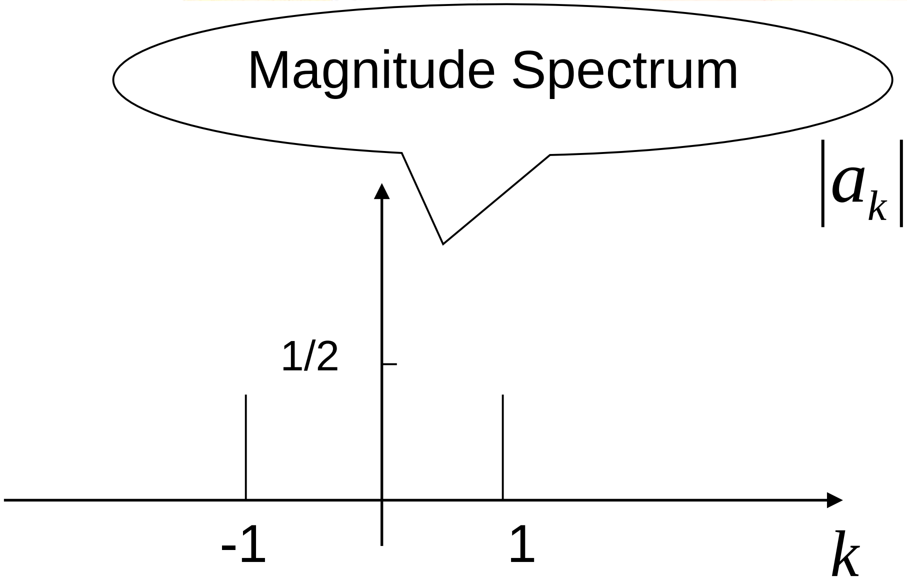


Example: $x(t) = \cos(\omega_0 t)$

$$\because x(t) = \cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$\therefore a_1 = a_{-1} = \frac{1}{2}$$

$$a_k = 0, \quad \mathrm{k} \neq \pm 1$$



Spectrum of x(t): Magnitude Spectrum, Phase Spectrum

How about： $x(t) = \sin(\omega_0 t)$

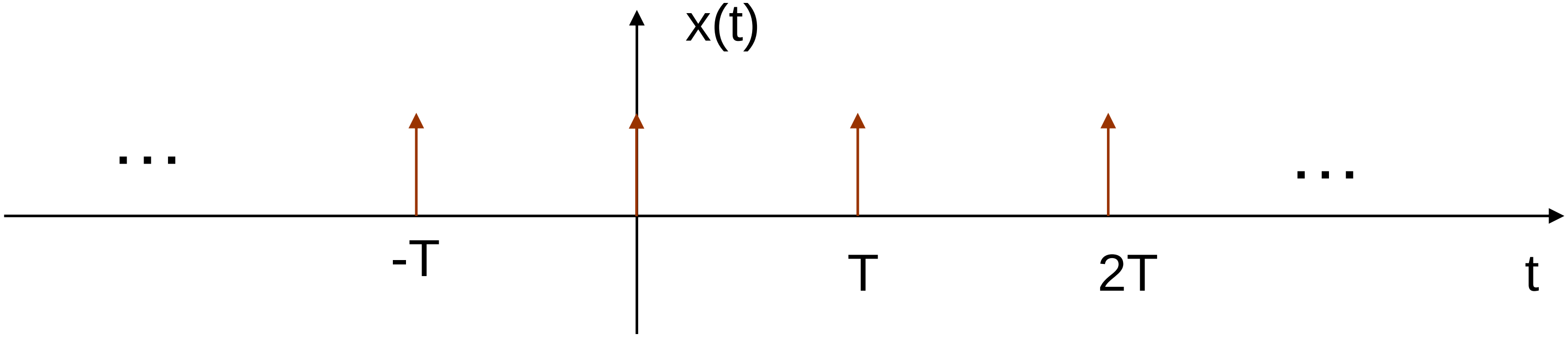
Observations:

1. The spectrum of the periodic signal x(t) is discrete, it has non-zero values only at $k\omega_0$, i.e. the spectrum space ω_0 is $2\pi/T$

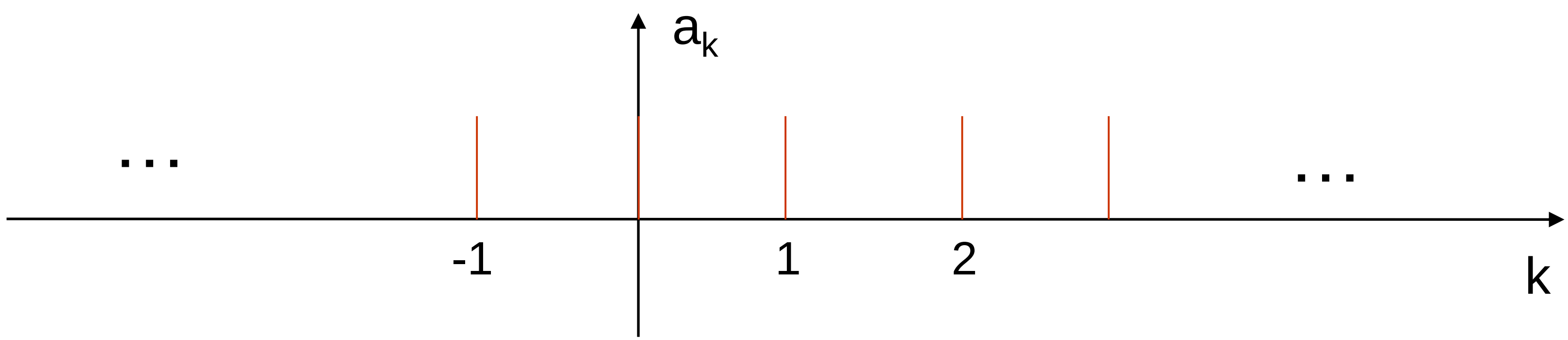
2. a_k is complex representing the magnitude and phase of k^{th} harmonic component

Notes: The negative frequencies (k<0) are meaningless in real world, they are for mathematical representations and derivations.

Example: $$x(t)=\sum_{k=-\infty}^{+\infty}\delta(t-kT)$$

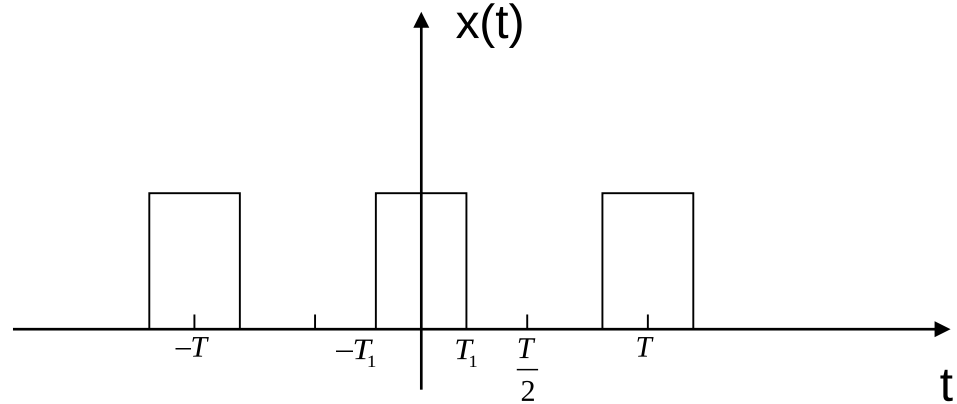


$$a_k=\frac{1}{T}\int_{-T/2}^{T/2}\delta(t)e^{-jk2\pi/Tt}dt=\frac{1}{T}$$



Ex: Periodic Square Wave

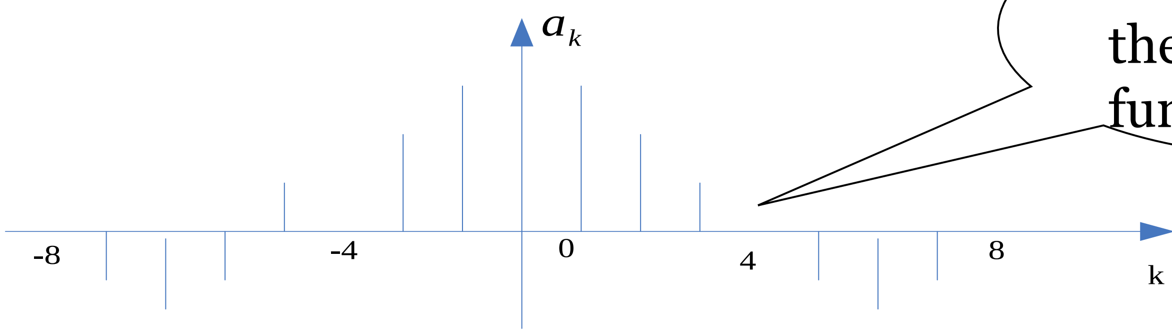
$$x(t)=\begin{cases}1, & |t|<T_1\\ 0, & T_1<|t|<T/2\end{cases}$$



$k=0$ $$a_0=\frac{1}{T}\int_{-T_1}^{T_1}dt=\frac{2T_1}{T}$$ ——DC

$k\neq 0$ $$a_k=\frac{1}{T}\int_{-T_1}^{T_1}1\cdot e^{-jk\omega_0 t}dt=\cdots=\frac{\sin(k\omega_0 T_1)}{k\pi}$$

Supposed $T=8T_1$



The envelop is the sampling function

F.C. of the periodic square wave with fundamental frequency of T and pulse width of $2T_1$:

$$\boxed{a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} = 2\frac{T_1}{T} Sa(k\omega_0 T_1)}$$

where the sampling function is defined as $Sa(x) = \frac{\sin(x)}{x}$

The first zero value of the F.C. of the periodic square wave with fundamental frequency of T and pulse width of $2T_1$ is at the k^{th} point satisfying $k\omega_0 T_1 = \pi$, i.e. the main lobe of the signal is $T/(2T_1)$ (Hz), i.e. the bandwidth of the signal

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Convergence of CT Fourier Series

- The key is: What do we *mean* by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad ?$$

- One useful notion for engineers: there is no *energy* in the difference

$$e(t) = x(t) - \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\int_T |e(t)|^2 dt = 0$$

(just need $x(t)$ to have finite energy per period)

$$\int_T |x(t)|^2 dt < \infty$$

Under a different, but reasonable set of conditions (the Dirichlet conditions)

Condition 1. $x(t)$ is *absolutely integrable* over one period, i. e.

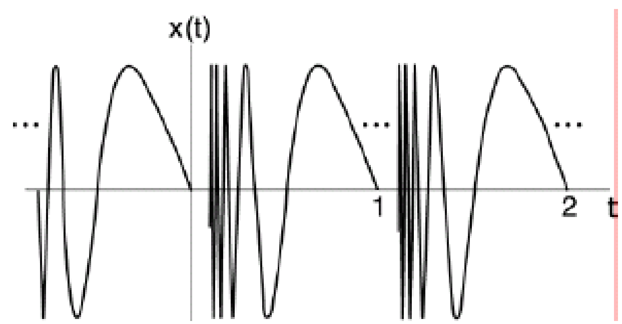
$$\int_T |x(t)|dt < \infty$$

And

Condition 2. In a finite time interval, $x(t)$ has a *finite* number of maxima and minima.

Ex. An example that violates Condition 2.

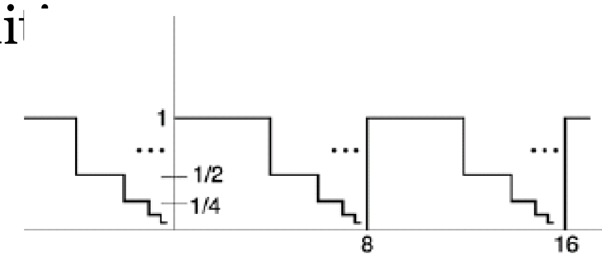
$$x(t) = \sin\left(\frac{2\pi}{t}\right) \quad 0 < t \leq 1$$



And

Condition 3. In a finite time interval, $x(t)$ has only a *finite* number of discontinui

Ex. An example that violates Condition 3.



- Signals do not satisfy the Dirichlet conditions are generally pathological in nature, and do not typically exist in real world.
-
- Dirichlet conditions are met for the signals we will encounter in the real world. Then

  - The Fourier series = $x(t)$ at points where $x(t)$ is continuous

  - The Fourier series = "midpoint" at points of discontinuity

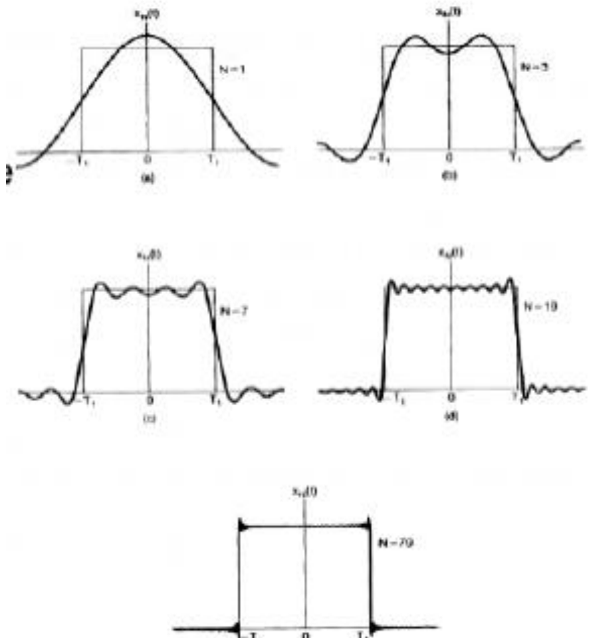
  - There has no energy difference between the original signal and its Fourier series representation

  - Since the original signal and its Fourier series representation only differ at isolated points, the integral of both signals over any interval are identical, i.e. the two signals behave identically under convolution, and during the analysis of LTI systems.

- Still, convergence has some interesting characteristics:

$$x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$$

-There exists error between the original signal x(t) and the approximation $x_N(t)$, it decreases as N increases
- As $N \rightarrow \infty$, $x_N(t)$ exhibits *Gibbs*' phenomenon at points of discontinuity (1.09)



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- **Linearity**

$$x(t) \leftrightarrow a_k \qquad y(t) \leftrightarrow b_k$$

$$\boxed{z(t) = Ax(t) + By(t) \leftrightarrow C_k = Aa_k + Bb_k}$$

- **Time Shifting**

$$x(t) \leftrightarrow a_k$$

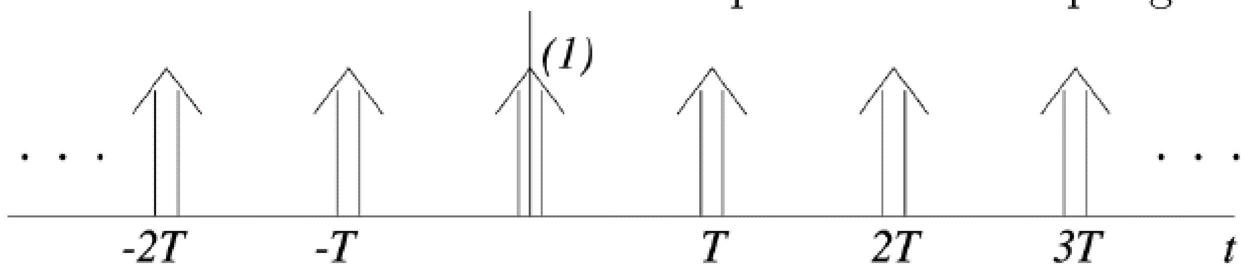
$$\boxed{x(t - t_0) \leftrightarrow e^{-jk\omega_0 t_0} a_k}$$

Time shifting introduces a linear phase shift $\propto t_0$ in frequency domain

Example: Periodic Impulse Train

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad - \quad \text{Sampling function}$$

$$\text{important for sampling}$$



$$\begin{aligned} a_k &= \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)e^{-jk\omega_0 t}dt \\ &= \frac{1}{T} \quad \text{for } all\ k \ ! \end{aligned}$$

$$\Downarrow$$

$$x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t}$$

— All components have:

(1) the same amplitude,

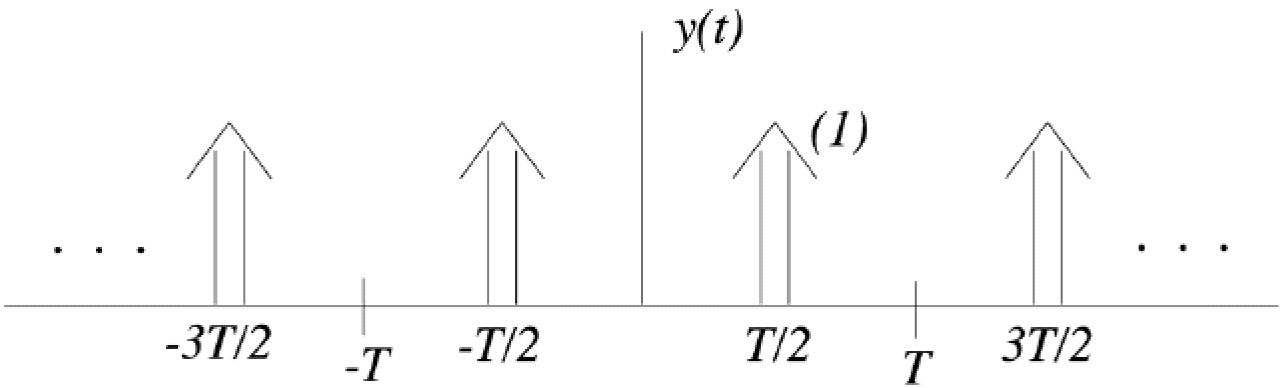
&

(2) the same phase.

Example: Shift by half period

$$y(t) = x(t - T/2) \;\leftrightarrow\; a_k e^{-jk\pi} = (-1)^k a_k$$

using $\quad e^{-jk\omega_0 T/2} = e^{-jk\pi}$



$$y(t) \leftrightarrow \underset{\displaystyle \frac{(-1)^k}{T}}{\underset{\|}{(-1)^k a_k}} \quad \left(a_k = \frac{1}{T} = \text{ F.C. of } \sum_{n=-\infty}^{\infty} \delta(t - nT) \right)$$

- **Freuency Shifting**

$$x(t) \leftrightarrow a_k$$

$$x(t)e^{jM\omega_0 t} \leftrightarrow a_{k-M}$$

Eg. Carrier Modulation

- **Time Reversal**

$$x(t) \leftrightarrow a_k$$

$$\boxed{x(-t) \leftrightarrow a_{-k}}$$

- **Time Scaling**

$$x(t) \leftrightarrow a_k$$

$$\boxed{x(\alpha t) \leftrightarrow a_k}$$

Compression of a signal in time domain results in spectrum expand

Although the F.C. of x(at) and x(t) are identical, they have different fundamental frequency

$$x(t) = \sum_k a_k e^{jk\omega_0 t} \qquad x(\alpha t) = \sum_k a_k e^{jk(\alpha\omega_0)t}$$

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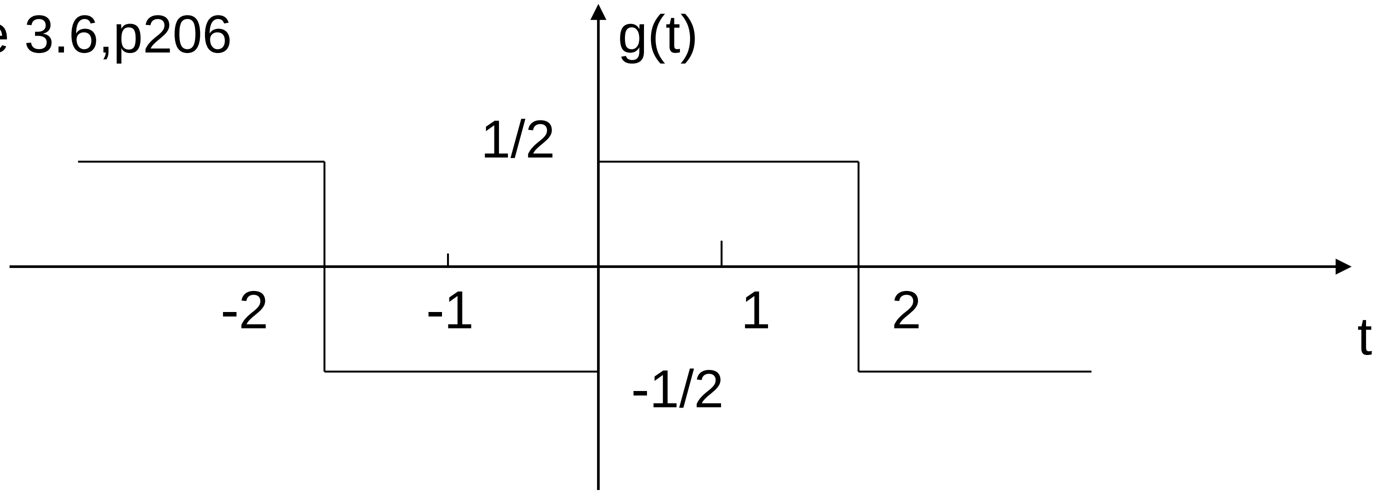
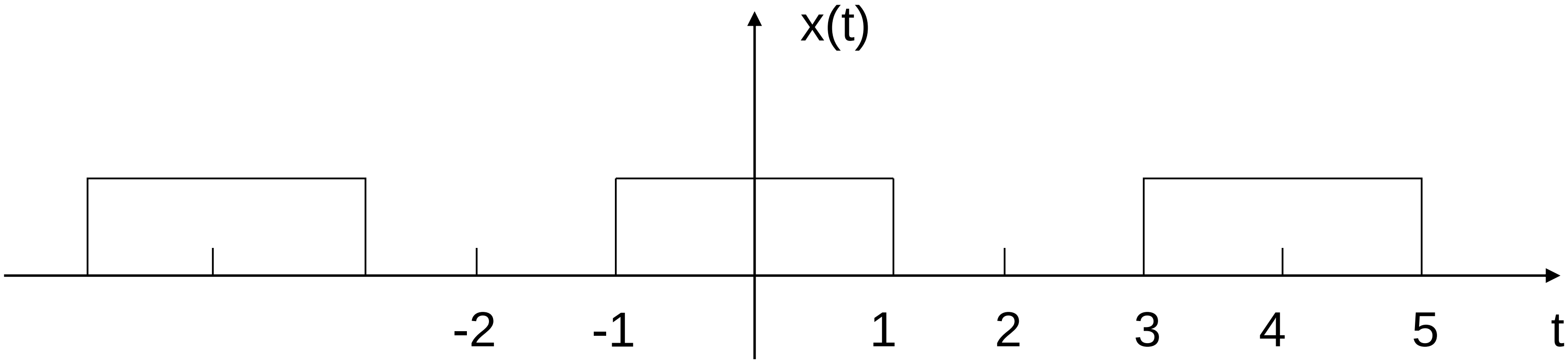




$$g(t) = x(t-1) - \frac{1}{2}$$

- **Differential Integral**

$$x(t) \leftrightarrow a_k$$

$$\boxed{\frac{dx(t)}{dt} \leftrightarrow jk\omega_0 a_k} \qquad \boxed{\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{jk\omega_0} a_k}$$

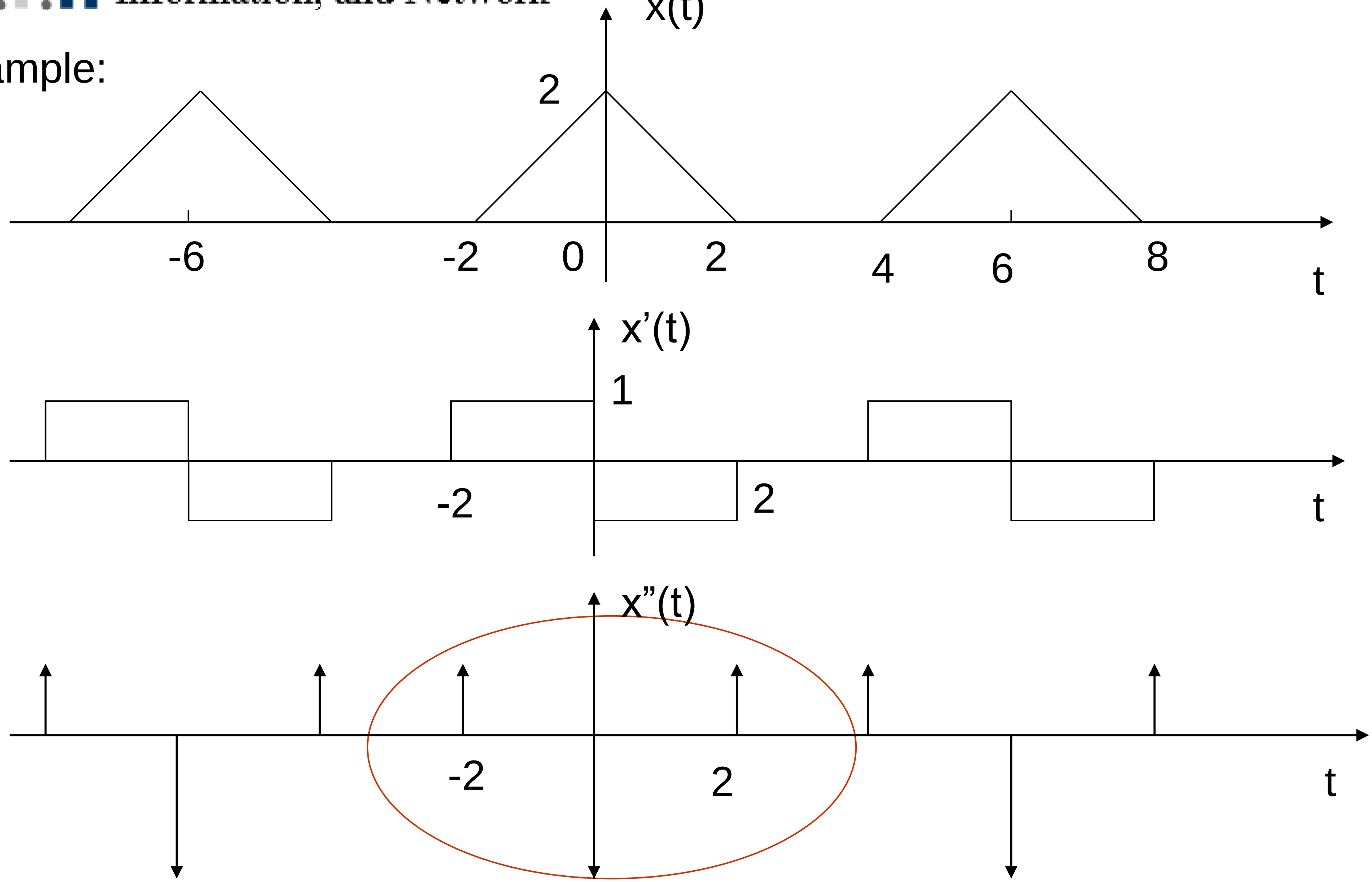
$$Note: \int_{-\infty}^{t} x(\tau)d\tau \ is \ finite \ valued \ and \ peridoic \ only \ if \ a_0 = 0$$

- **Multiplication**

$$x(t) \leftrightarrow a_k \qquad y(t) \leftrightarrow b_k$$

$$\boxed{x(t) \cdot y(t) \leftrightarrow \sum_{l=-\infty}^{\infty} a_l b_{k-l}}$$

Example:



• **Conjugation and Conjugate Symmetry**

$$x(t) \leftrightarrow a_k$$

$$\boxed{x^*(t) \leftrightarrow a^*_{-k}}$$

If x(t) is real $\qquad a_k = a^*_{-k}$ --Conjugate Symmetry

$$\mathrm{Re}\{a_k\} = \mathrm{Re}\{a_{-k}\}$$

$$\mathrm{Im}\{a_k\} = -\mathrm{Im}\{a_{-k}\}$$

$$|a_k| = |a_{-k}|$$

$$\angle a_k = -\angle a_{-k}$$

$$x(t)\ real\ and\ even \leftrightarrow a_k\ real\ and\ even$$

$$x(t)\ real\ and\ odd \leftrightarrow a_k\ purely\ imaginary\ and\ odd$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \leftrightarrow \mathrm{Re}\{a_k\}$$

$$x_0(t) = \frac{1}{2}[x(t) - x(-t)] \leftrightarrow j\,\mathrm{Im}\{a_k\}$$

•Parseval's Relation

$$\frac{1}{T}\int_{T}|x(t)|^{2}dt=\sum_{k=-\infty}^{\infty}|a_{k}|^{2}$$

$\frac{1}{T}\int_{T}|x(t)|^{2}dt$ _average power in one period of x(t)

$|a_{k}|^{2}$ _average power in the *k*th harmonic component

Energy is the same whether measured in the time-domain or the frequency-domain

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Fourier Series Representation of DT Periodic Signals

- $x[n]$ -periodic with fundamental period N, fundamental frequency

$$x[n+N] = x[n] \quad \text{and} \quad \omega_0 = \frac{2\pi}{N}$$

- Only $e^{j\omega n}$ which are periodic with period N will appear in the *FS*

$$\omega N = k2\pi \Leftrightarrow \omega = k\omega_0 \quad , \quad k = 0, \pm 1, \pm 2, \cdots$$

- There are only N distinct signals of this form

$$e^{j(k+N)\omega_0 n} = e^{jk\omega_0 n} e^{j\overbrace{N\omega_0 n}^{2\pi n}} = e^{jk\omega_0 n}$$

- So we *could* just use only N distinct exponential sequenceus to represent a DT periodic signal
- However, it is often useful to allow the choice of N consecutive values of k to be *arbitrary*.

DT Fourier Series Representation

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$

$\sum_{k=<N>}$ **= Sum over *any N* consecutive values of *k***

— This is a *finite* series

$\{a_k\}$ **- Fourier (series) coefficients**

Questions:

1) **What DT periodic signals have such a representation?**
2) **How do we find a_k?**

Answer to Question #1:

Any DT periodic signal has a Fourier series representation

$$
\begin{aligned}
x[n] &= \sum_{k=<N>} a_k e^{jk\omega_0 n} \\
&\Downarrow \\
x[0] &= \sum_{k=<N>} a_k \\
x[1] &= \sum_{k=<N>} a_k e^{jk\omega_0} \\
x[2] &= \sum_{k=<N>} a_k e^{j2k\omega_0} \\
&\vdots \\
x[N-1] &= \sum_{k=<N>} a_k e^{j(N-1)k\omega_0}
\end{aligned}
$$

N equations for N unknowns, a_0, a_1, ..., a_{N-1}

A More Direct Way to Solve for a_k

Finite geometric series

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & ,\alpha = 1 \\ \dfrac{1-\alpha^N}{1-\alpha} & ,\alpha \neq 1 \end{cases}$$

$$\Downarrow \qquad \alpha = e^{jk\omega_0}$$

$$\sum_{n=<N>} e^{jk\omega_0 n} = \sum_{n=0}^{N-1} \left(e^{jk\omega_0}\right)^n = \sum_{n=0}^{N-1} \left(e^{jk2\pi/N}\right)^n$$

$$= \begin{cases} N & , k = 0, \pm N, \pm 2N, \cdots \\ \dfrac{1-e^{jk(2\pi/N)N}}{1-e^{jk\omega_0}} = 0 & , \text{ otherwise} \end{cases}$$

So, from $$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}$$

multiply both sides $e^{-jm\omega_0 n}$

and then $\sum_{n=<N>}$

$$\sum_{n=<N>} x[n]e^{-jm\omega_0 n} = \sum_{n=<N>} \left(\sum_{k=<N>} a_k e^{jk\omega_0 n} \right) e^{-jm\omega_0 n}$$

$$= \sum_{k=<N>} a_k \underbrace{\left(\sum_{n=<N>} e^{j(k-m)\omega_0 n} \right)}_{=N\delta[k-m]-}$$

orthoronality

$$= N a_m$$

$$\Downarrow$$

DT Fourier Series Pair

$$\left(\omega_0 = \frac{2\pi}{N}\right)$$

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} \qquad \text{(Synthesis equation)}$$

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} \qquad \text{(Analysis equation)}$$

Note:It is convenient to think of a_k as being defined for *all* integers k. So:

1) $a_{k+N} = a_k$ **—Special property of DT Fourier Coefficients.**

2) **We only use N consecutive values of a_k in the synthesis equation. (Since $x[n]$ is periodic, it is specified by N numbers, either in the time or frequency domain)**

Example #1: Sum of a pair of sinusoids

$$x[n] = \cos(\pi n/8) + \cos(\pi n/4 + \pi/4)$$

periodic with period $= 16 \Rightarrow \omega_0 = \pi/8$

$$x[n] = \frac{1}{2}\left[e^{j\omega_0 n} + e^{-j\omega_0 n}\right] + \frac{1}{2}\left[e^{j\pi/4}e^{j2\omega_0 n} + e^{-j\pi/4}e^{-j2\omega_0 n}\right]$$

$$\Downarrow$$

$$\begin{aligned}
a_0 &= 0 \\
a_1 &= 1/2 \\
a_{-1} &= 1/2 \\
a_2 &= e^{j\pi/4}/2 \\
a_{-2} &= e^{-j\pi/4}/2 \\
a_3 &= 0 \\
a_{-3} &= 0 \\
&\vdots
\end{aligned}$$

$$a_{15} = a_{-1+16} = a_{-1} = 1/2$$

$$a_{66} = a_{2+4\times 16} = a_2 = e^{j\pi/4}/2$$

Homework

BASIC PROBLEMS WITH ANSWER:

3.3, 3.4

BASIC PROBLEMS:

3.22(a), 3.23